An Alternate Approach to Creep-Fatigue Damage with Elastic Follow-up for High Temperature Structural Design

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The Current ASME code, subsection NH, and the Japanese High Temperature Structural Design Guide are based on the separation of elevated temperature cyclic damage into two parts, creep damage and fatigue damage. This presents difficulties in both the evaluation of test data and the determination of cyclic damage in a design. To avoid these difficulties, R.I. Jetter proposes an alternate approach. The test is designed to have follow-up characteristics conservatively bounding the follow-up in the general structural components of interest; thus, the actual stress/strain in the test is greater than in the component for the same elastically calculated peak stress/strain. The authors performed creep-fatigue tests with elastic follow-up at 550°C using SUS304 stainless steel specimens. The reduction characteristics of the creep-fatigue damage with elastic follow-up at 550°C are obtained.

1 Elastic Follow-up Characteristics

1.1 Examples of elastic follow-up phenomena

There are two areas of the ASME Code which require consideration of elastic follow-up⁶. One area is the consideration of secondary stresses with a "large amount of elastic follow-up" as a load controlled quantity. The only definition of elastic follow-up currently in the ASME Code is a modified version of the discussion in the Power Piping Code, B31-1, on local overstain. In practice, piping is the most common application of the current definition and the term "large amount of elastic follow-up" would usually apply to an analysis of a complete pipe line.

It is difficult to describe quantitatively what is a large amount of elastic follow-up. In the context of the stated rules for load controlled stresses, which are largely based on the results of Robinson⁹ and others, the intent is to consider a restrained thermal expansion stress in systems with localized weak areas, such as Fig.1. However, restrained thermal expansion in a well-balanced system, as represented by Fig.2, should not be considered primary.

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Fig. 1. Example of a restrained thermal expansion stress in a system with localized weak area.

Fig. 2. Example of a restrained thermal expansion in a well-balanced system.
The other part of the Code which considers elastic follow-up is in the Appendix T rules for deformation controlled quantities\(^9\), and relates to the application of elastic analysis to the satisfaction of strain limits.

Stepped bars are considered in series subjected to a displacement \(\delta\) as shown in Fig.3 through Fig.5\(^9\). The stresses and strains associated with the displacement \(\delta\) would nominally be considered deformation controlled quantities. However, depending upon the relative stiffness of the stepped bars, the stresses in each will require separate interpretation of the Code rules.

In Fig.3, the area \(a_2\) of Bar 2 is very much larger than \(a_1\) of Bar 1. The mutual deflection \(\delta_1\) and \(\delta_2\) of Bar 1 and 2 are calculated elastically.

\[
\delta_1 = F_1/k_1 = F_1l_1/Ea_1,
\]
\[
\delta_2 = F_2/k_2 = F_2l_2/Ea_2
\]

(1)

where \(Fi\) = axial force, \(k_i\) = spring constant, \(l_i\) = bar length, \(i = 1\) or 2, and \(E\) = elasticity.

Since the deflection \(\delta_1\) and \(\delta_2\) must be added up to the total applied deflection \(\delta\),

\[
\delta = \delta_1 + \delta_2
\]

(2)

and the applied axial force \(F_1 = F_2\).

\[
a_1E\delta_1/l_1 = a_2E\delta_2/l_2
\]

Therefore, the following equation is obtained,

\[
\delta = (1 + a_1l_2/a_2l_1)\delta_1
\]

(3)

If \(a_1 \ll a_2\) and \(l_2\) is nearly equal to \(l_1\), \(a_1l_2/a_2l_1\) is negligible small. This means that spring constant \(k_2\) is very much larger than \(k_1\). In due course,

\[
\delta \approx \delta_1
\]

namely all the deformation will take place in 1. This is analogous to a local thermal stress such as a small hot spot in a vessel wall. In essence, there is no elastic follow-up because the displacement-induced load in 1 causes a negligible change in the displacement of 2. Therefore, the resultant load history in 1 can be considered solely on the basis of creep-fatigue.

In Fig.4, Bar 1 and 2 are of equal area and length, and operate at the same temperature. When subjected to a displacement \(\delta\), the bars deflect an equal amount and this displacement does not change even if yielding or creep takes place. Thus, there is no elastic follow-up. This case is analogous to the stresses produced in a vessel wall due to a radial thermal gradient. Thus, stresses due to radial temperature variations through the wall are specifically exempted from the restriction that secondary stresses must be considered as primary stress in applying the strain limit evaluation technique, of which intent is to consider displacement-induced stresses with elastic follow-up as primary stress in applying the strain limit evaluation.

In Fig.5, where there is elastic follow-up, the area \(a_1\) is smaller than \(a_2\) but still strong enough to cause an initial deflection \(\delta_2\) in 2.

First, the mutual deflections of 1 and 2 is considered if the elastically calculated stress in 1 exceeds its yield strength. The elastically calculated deflections of 1 and 2 will be as shown; however, since the actual stress in 2...
will not exceed its yield strength, the actual load and resultant deflection in $\varnothing$ will be less than the elastically calculated load and deflection. Since the deflection $\delta_1$ and $\delta_2$ must be added up to the total applied deflection $\delta$, the deflection $\delta_1$ will be greater than elastically calculated. This is an example of elastic follow-up.

Next, consider the relative deflections of $\varnothing$ and $\varnothing$ assuming that creep is taking place; the area of $\varnothing$ is smaller than $\varnothing$ and initial deflection of $\varnothing$ is significant with respect to $\varnothing$. Since the stress in $\varnothing$ is greater than the stress in $\varnothing$, the stress in $\varnothing$ will relax at a faster rate. As the stress in $\varnothing$ relaxes, the stress and deformation of $\varnothing$ will decrease tending toward its unloaded position, and the deformation of $\varnothing$ will increase; again this is elastic follow-up. If there is a large amount of elastic follow-up, the load on $\varnothing$ will be practically constant and can be considered as primary stress even though the initial source of the load was a displacement. The presence of elastic follow-up thus results in a more constant load, slower stress relaxation, and more strain accumulation as compared to cases where there is no elastic follow-up.

![Diagram](image-url)

**Fig. 3.** Elastic Deflection when $a_2 \gg a_1$

![Diagram](image-url)

**Fig. 4.** Elastic Deflection when $a_2 = a_1$
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In another example, it is considered the stresses produced by the temperature difference between a nozzle and the shell to which it is attached. This can be idealized as a built-in cylinder as shown in Fig. 6. The initial elastically calculated deflection curve is shown as 0 - A. If there were no moment resistance at the built-in joint, then it would behave structurally as a pinned joint as shown by curve 0 - D. If the initial stress at the built-in end exceeds yield, then a partial plastic hinge will result with an increase in strain at the joint as shown by 0 - B. If the joint creeps, there will be further strain redistribution as shown by curves 0 - C. In effect, the angular rotation of the relatively localized high stress at the joint is being driven by the lower stressed, beam-on-elastic foundation behavior of the cylinder. Thus, the built-in cylinder is an example of elastic follow-up.

If one were to apply the results of the elastic analysis directly, as in the case of stresses generated by a radial temperature gradient, then the surface strain at the built-in cylinder would be underestimated. This is the reason for the restriction on the application of the strain limit evaluation technique which states that any secondary stress with elastic follow-up must be considered as primary stress for purposes of that evaluation. This is conservative in that it assumes that the built-in cylinder effect will have a large amount of stored elastic energy.
Fig. 6. Elastic follow-up in built-in cylinder

O-D: Pinned joint curve

O-C: Joint creep curve
due to elastic follow-up

O-B: Partial plastic hinge curve
due to elastic follow-up

O-A: Initial elastically calculated
deflection curve

Temperature

$T_1 > T_0$
1.2 Stress-strain behaviors and elastic follow-up factor $q_n$

When a tensional force is loaded on the flat bar, the stress and strain increases elastically from point 0 to A in Fig.7. If the displacement is kept constantly at point A, the stress-strain changes from point A to C (Displacement-controlled behavior). But, if the load is kept constantly at the point A, the stress-strain changes from A to B (Load-controlled behavior).

However, when the total displacement of stepped bar is kept constantly at the point A, the stress-strain changes from A to D due to the elastic follow-up phenomenon.

The elastic follow-up factor $q_n$ is defined by Eq. (4)^−7).

\[
q_n = \frac{\Delta \epsilon - \Delta \sigma / E}{- \Delta \sigma / E} = \frac{Y}{X}
\]

(4)

where X and Y are shown in Fig.7 and $\Delta \sigma < 0.$ In the case of stepped bar model of Fig.8, the elastic follow-up factor $q_n$ is obtained as follows.

As the total elongation $\delta$ is consisted of $\delta_1$ of the small diameter bar and $\delta_2$ of the large diameter bar,

\[
\delta = \delta_1 + \delta_2 = \delta_1 + \delta_2
\]

\[
\frac{d\delta}{dt} = 1_1 \delta_1 + 1_2 \delta_2 = 0
\]

(5)

and the tensional force is the same at each section,

\[
a_1 \delta_1 = a_2 \delta_2
\]

(6)

On the other hand, the total strain $\epsilon$ consists of the elastic strain $\epsilon_e$ and the creep strain $\epsilon_c$.

\[
\epsilon_1 = \epsilon_e + \epsilon_c = \frac{\delta_1}{E} + \epsilon_c
\]

\[
\dot{\epsilon}_1 = \frac{\delta_1}{E} + \epsilon_c = \frac{\delta_1}{E} + B \sigma_i^m
\]

(7)

\[
\epsilon_2 = \epsilon_e + \epsilon_c = \frac{\delta_2}{E} + \epsilon_c
\]

\[
\dot{\epsilon}_2 = \frac{\delta_2}{E} + \epsilon_c = \frac{\delta_2}{E} + B \sigma_i^m
\]

(8)

where the Norton-Bailey's equation is applied for the creep strain rate ($\dot{\epsilon}_c$) and the factors of $B$ and $m$ are the material constants.

From Eq.(6) and Eq.(8),

\[
\dot{\epsilon}_2 = \left( \frac{a_1}{a_2} \right) \frac{\dot{\epsilon}_1}{E} + B \left( \frac{a_1}{a_2} \right)^m \sigma_i^m
\]

(9)

then Eq.(7) and Eq.(9) are put into Eq.(5),

\[
\frac{\dot{\delta}}{E} \left( 1 + \frac{1_1}{1_2} \left( \frac{a_1}{a_2} \right) \right) = - \left( 1 + \frac{1_1}{1_2} \left( \frac{a_1}{a_2} \right)^m \right) B \sigma_i^m
\]

\[
\dot{\delta}_1 = - \left( \frac{1 + \left( 1_2 / 1_1 \right) \left( a_1 / a_2 \right)^m}{1 + \left( 1_2 / 1_1 \right) \left( a_1 / a_2 \right)^m} \right) E \sigma_i^m
\]

(10)

From Eq.(7) and Eq.(10),

\[
\dot{\epsilon}_1 = \left( 1 - \frac{1 + \left( 1_2 / 1_1 \right) \left( a_1 / a_2 \right)^m}{1 + \left( 1_2 / 1_1 \right) \left( a_1 / a_2 \right)^m} \right) B \sigma_i^m
\]

(11)

Eq.(4) is transformed,

\[
q_n = \frac{E \cdot \frac{\Delta \epsilon_1 - \Delta \sigma_1}{\Delta t}}{\frac{\Delta \sigma_1}{\Delta t}} = \frac{E \cdot \dot{\epsilon}_1 - \dot{\delta}_1}{\dot{\delta}_1}
\]

(12)

then, Eq.(10) and Eq.(11) are put into Eq.(12).

Finally, $q_n$ is obtained.
\[ q_n = \frac{1 + (l_2/l_1)(a_2/a_1)}{1 + (l_2/l_1)(a_2/a_1)^m} \]  

Eq. 13 shows that the stress relaxation speed decreases by \( q_n = 1.0 \), if the structure has no elastic follow-up behavior at all.

\[ \dot{\epsilon}_1 = \frac{1}{q_n} E \beta \sigma_1^m \]  

Eq. 14

Fig. 7. Comparison of stress-strain behaviors among Load-Controlled Condition, Displacement-Controlled condition, and Elastic Follow-up Condition

Fig. 8. Stepped bar model
2 Testing Conditions

2.1 Methodology of the alternate approach

R.I.Jetter proposes an alternate approach for the life prediction of the creep-fatigue damage with the elastic follow-up\(^a\). In the proposed approach, cyclic damage is not separated into a creep component and a fatigue component as ASME code, Subsection NH. Instead, the effects of strain redistribution and the combined effects of creep and fatigue damage can be accounted for in a simplified model test suitably sized to ensure the cyclic damage in the test bounds the damage in an actual structure when subjected to comparable loading. The test article is a displacement controlled hold time creep-fatigue specimen with elastic follow-up. The measure of loading comparability is the maximum elastically calculated strain range in the component.

The component of a hypothetical stepped cylinder has a global elastic follow-up, \(q_n\). The damage from a strain, \(\varepsilon_F\), which is applied, held, and then cycled back to zero and reapplied is evaluated from a design curve based on data from the test. The evaluation procedure is envisioned to be essentially the same as that used in Subsection NB of ASME Sec. III\(^a\) and Japanese Notice No.501\(^a\) where the damage fraction is determined as the ratio of actual number of applied cycles, \(n\), to the allowed number of cycles, \(N\), with the same range, \(\varepsilon_F\).

Ideally, for a given temperature there would only be two design curves, one for short term loads, such as seismic, without a hold time effect, and another curve which envelops the effects of hold time duration and follow-up magnitude without excessive conservatism.

The design curve is to be developed from the test data which is plotted as elastically calculated strain vs. observed cycles to failure. The specimen is sized to envelope the follow-up characteristics of interest. A design margin must be applied to the data.

The test specimen must be sized to provide a stress strain histogram under cyclic loading which envelops the histogram of the components of interest.

2.2 Used materials and tested temperature

As Stainless Steel of SUS304 (JIS G 4308-1991) is used most generally in the conventional elevated temperature components such as heat exchangers, pressure vessels etc, SUS304 is selected for the materials of the test specimen, of which chemical compositions are shown in Table 1. All tests are performed at the constant temperature of 550 °C by using hydraulic servo creep fatigue testing machine. The test temperature 550°C is chosen due to the remarkable creep phenomenon at 550°C and to be very close to the maximum design temperature. Number of cycles to failure \(N_f\) is accounted by the separation of the specimen.
Table 1. Chemical compositions of SUS304(JIS G 4308-1991)

<table>
<thead>
<tr>
<th>C</th>
<th>S</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Ni</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.08</td>
<td>&lt;1.00</td>
<td>&lt;2.00</td>
<td>&lt;0.045</td>
<td>&lt;0.030</td>
<td>8.00~10.50</td>
<td>18.00~20.00</td>
</tr>
</tbody>
</table>

2. 3 Decision of the dimensions of the test specimens.

Stepped bar of the test specimen is modeled preliminary for the elastic follow-up as shown in Fig.8. The elastic follow-up factor \( q_n \) can be obtained from Eq.(13). The relationships between the area ratio \( a_1/a_2 \) and the elastic follow-up factor \( q_n \) are calculated at the parameter of length \( l_1 \) by keeping the total length \( l = 120 \text{mm} \) constant, as shown in Fig.9.

This figure shows the maximum value of \( q_n \) takes place when ratio \( a_1/a_2 \) is around 0.5. As the maximum value of \( q_n \) of the practical components is mentioned about 2.0\(^1\), the ideal configuration is confirmed to be very close to the normally used creep-fatigue test specimen. Authors chose the dimensions of that specimen as the practical test specimen, of which configuration is shown in Fig.10. The area ratio \( a_1/a_2 \) is 0.444, \( l_1 \) is 47 mm and \( l_2 \) is 73 mm. From the Eq.(13), \( q_n = 1.64 \) is obtained when \( m = 5 \). The sectional diameter is changed smoothly by the radius of curvature \( R = 35 \text{mm} \) as shown in Fig.10, where the stress concentration factor \( K = 1.00 \) is calculated from the stress concentration factor chart at the tensional load\(^{40} \).

3 Test Results

3. 1 Elastic characteristic test of the test specimen

Since the obtained test data are to be used for the life prediction with the elastic follow-up on the basis of elastic stress analysis, the test data must be arranged in the cycles to failure \( N_t \) vs. the elastically calculated strain \( \varepsilon_{e,x} \). The elastically calculated strain must be consistent to the strain of the elastic stress analysis.

A test specimen, shown in Fig.10, is extended by the stroke control. Both elongation of \( \delta \text{ (mm)} \) for the total length 120 mm (stroke control length) and \( \lambda_1 \text{ (mm)} \) for the gauge length 15 mm are measured by each extension step. From the plots of \( \delta \) and \( \lambda_1 \) vs. load \( F \text{ (kN)} \), elastic characteristics are obtained,

\[
\delta = 0.01F, \quad \lambda_1 = 0.0013F
\]
as shown in Fig. 11. Accordingly, the elastically calculated strain $\varepsilon_{ce}$ can be obtained by Eq. 16.

$$\varepsilon_{ce} = \frac{\lambda_1}{(\text{gauge length})} = \frac{\lambda_1}{15} = 8.67 \times 10^{-3} \delta$$ (16)

In the case of the strain controlled tests,

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![Diagram](image_url)

**Fig. 9.** $\sigma_a$ curve in the case of stepped bar model by keeping total length $l_1=120$mm

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![Diagram](image_url)

**Fig. 10.** Test specimen configuration
3.2 Elastic follow-up test results

The obtained data are shown in Table 2 and Fig.12.

$N_f$ of the strain controlled tests with 600 sec. hold time is lower than that of the strain controlled tests with no-hold time, where the gauge length for the strain control was 15mm. This comparison is very similar to the normal creep fatigue tests. However, $N_f$ of the stroke controlled tests with no-hold time decreases more from the strain controlled data. It is thought that the effect of the elastic follow-up has been caused. $N_f$ of the stroke controlled tests with 600 sec. Hold time shows to decrease much more.

<table>
<thead>
<tr>
<th>$\delta$ (mm)</th>
<th>$\varepsilon_{ce}$ (%)</th>
<th>$\varepsilon_c$ (%)</th>
<th>$N_f$ (Strain control)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1910</td>
<td>1059</td>
</tr>
<tr>
<td>0.85</td>
<td>0.85</td>
<td>4200</td>
<td>2100</td>
</tr>
<tr>
<td>0.70</td>
<td>0.70</td>
<td>4300</td>
<td>3933</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
<td>(1)56001</td>
<td>(2)85000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta$ (mm)</th>
<th>$\varepsilon_{ce}$ (%)</th>
<th>$\varepsilon_c$ (%)</th>
<th>$N_f$ (Stroke control)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20</td>
<td>1.04</td>
<td>1</td>
<td>320</td>
</tr>
<tr>
<td>1.02</td>
<td>0.88</td>
<td>0.85</td>
<td>435</td>
</tr>
<tr>
<td>0.84</td>
<td>0.73</td>
<td>0.70</td>
<td>1350</td>
</tr>
<tr>
<td>0.60</td>
<td>0.62</td>
<td>0.50</td>
<td>3000</td>
</tr>
</tbody>
</table>
4 Discussions

4.1 Reduction of $N_f$ due to the Elastic follow-up

From the tests, each best fit curve is obtained for the global elastic follow-up evaluation as shown in Fig.12. At first, the curve of the strain control test with 600 sec. hold time is confirmed to reduce much more than the curve of the strain control with no-hold time.

The curve of the stroke control with no-hold time reduces to approximate $1/10$ of $N_f$ from that of the strain control test with no-hold time. And the curve of the stroke control with 600 sec. hold time reduces much more than the stroke control test with no-hold time.

Those reduction of $N_f$ is thought to be caused by the elastic follow-up. A design margin must be applied to the data to account for such factors as data scatter and extrapolation to longer hold times and smaller total strain amplitudes, but both curves of the stroke controlled tests show the
applicability to the creep fatigue evaluation with a global elastic follow-up, if the elastically calculated strain rate is applied to the curves. The curve of the stroke controlled tests with no-hold time is for the short term loads such as thermal transient, earthquake etc. And the curve of the stroke controlled tests with hold time is for the normal operating loads with hold time.

4.2 Hold Time Considerations

From the aspect of simplifying the design procedure, it would be desirable to have two curves at each temperature for cyclic damage evaluation, one curve for short time loading such as seismic and one other for all non short time events.

Hold time tests at 600sec. by the stroke control were carried out at 550°C and the results are compared with the no-hold time tests at the same strain rate as shown in Fig.12. Each N_t number shows to reduce a factor of approximate 1/2 due to the creep damage at 600sec. hold time. Considering a representative number of significant events over the life time of a plant, a reference hold time of 1000hr does not seem unreasonable. However, such a hold time is clearly not attainable directly from testing.

One approach to the problem is to extrapolate shorter hold time data to longer hold times assuming that cyclic life continues to decrease at the same exponential rate with hold time. A review of hold time creep-fatigue data (without elastic follow-up) is suggested that the cycle life reduces a factor of 2/3 to 1/3 for each factor of 10 increase in hold time\(^{10}\). The cycle life would normally be reduced about factor of 10 in extrapolating to a hold time of 1000hr. Therefore, the obtained best fit curves is extrapolated to 1000hr hold time for practical design.

5 Summary

The simplified model tests of SUS304 were performed for an alternate approach to the creep fatigue evaluation of an elevated temperature structural design. The obtained best fit curves shows the effect of the elastic follow-up and the feasibility of the application to the practical design evaluation on the basis of the elastic analyses.

From the test results, the expected best fit curves are obtained for the creep-fatigue damage evaluation with the elastic follow-up at 550°C for the elastic follow-up factor \(q_n = 1.64\). Those two curves shows feasible to apply the evaluation of the global structure with elastic follow-up if \(q_n\) of the structure is less than 1.64.

In future, a design margin must be applied to the curves to account for safety margins. And tests for more than \(q_n = 1.64\) shall be also performed.

Acknowledgment

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8) ASME, Boiler and Pressure Vessel Code, Section III, Division 1, Subsection NB, (1995).


高温構造設計における弾性追従を伴うクリープ・疲労損傷の
代替評価法研究

小畑清和・太田博光・石山英樹・貝原 亨・上野 修

現在のASME基準や日本の高温設計指針は、高温における繰返し損傷をクリープ損傷と疲労損傷の二つに分け
て評価しているが、複雑で難解なものとなっている。この困難を避ける為、R.I.Jetterは弾性追従を伴う場合の評
価を安全サイドにカバーする簡易方法を提案している。本研究では、汎用ステンレス鋼 SUS304 の試験片を用い、
550℃で提案の試験を行い、実用性のある特性線図を求めた。